

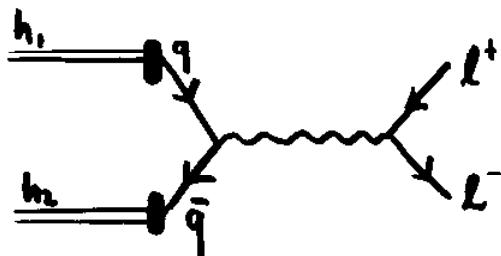
# QCD corrections to the polarized Drell-Yan process

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# in hadron-hadron collisions

## Parton model interpretation:



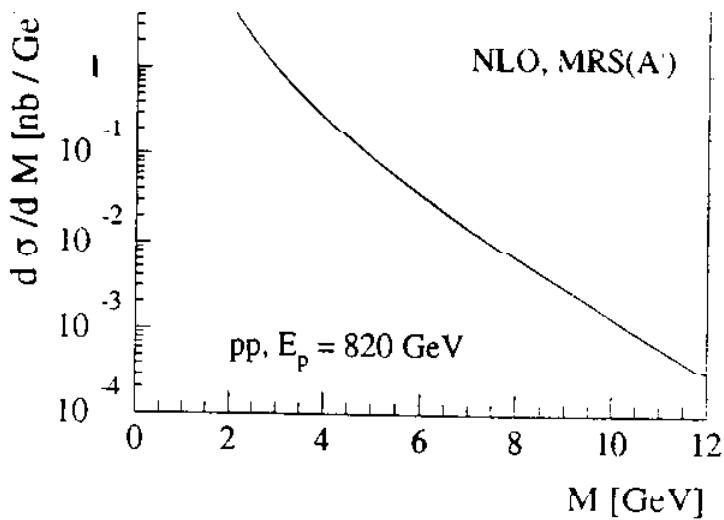
(at collider energies  
 $\gamma^* \rightarrow e^+, e^-$

$$\frac{dG}{dM^2} (h_1 + h_2 \rightarrow l^+ l^- + X)$$

$$= \sum_q \int dx_1 dx_2 [q_{h_1}(x_1) \bar{q}_{h_2}(x_2) + q_{h_2}(x_2) \bar{q}_{h_1}(x_1)] \frac{d\hat{G}}{dM^2}^{q\bar{q} \rightarrow l^+ l^-}(x_1, x_2, S)$$

$$\text{with: } \frac{d\hat{G}}{dM^2}^{q\bar{q} \rightarrow l^+ l^-} = \frac{4\pi\alpha^2}{g_{x_1 x_2} S} e_q^2 \delta(x_1 x_2 S - M^2)$$

- clear experimental signature in hadronic environment
- in pN collisions: probe of the antiquark distributions in the nucleon



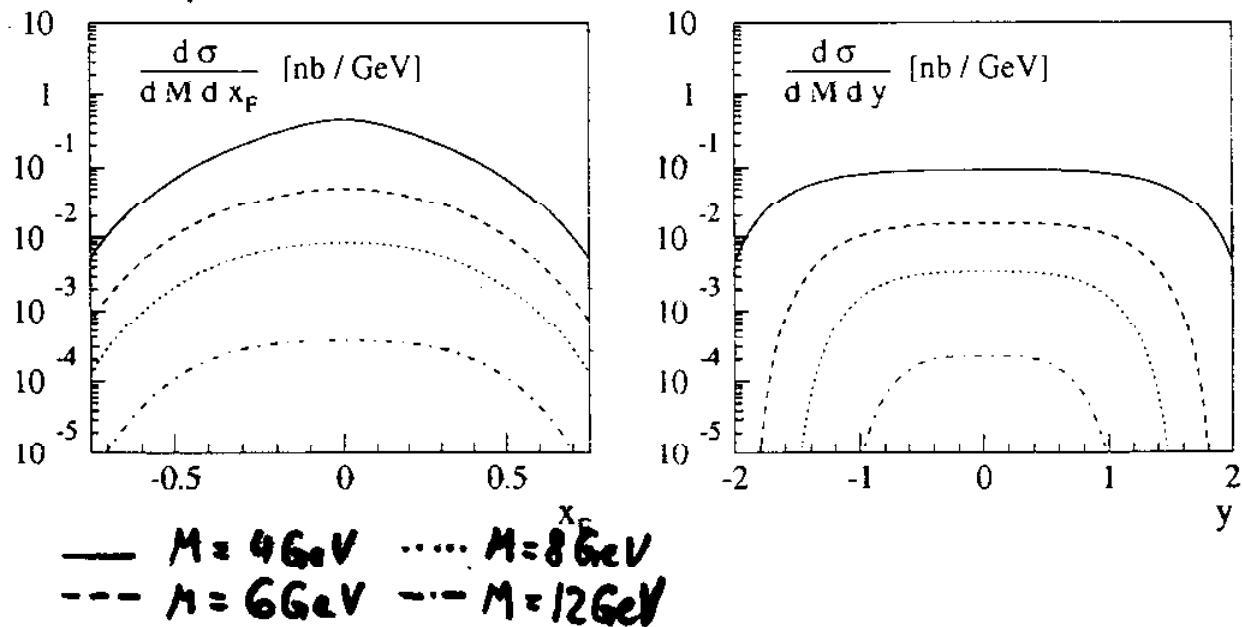
More differential information on beam and target structure is gained from distributions in :

$$x_F = \frac{2q_2}{\sqrt{s}}$$

$$\underset{\text{parton}}{\approx} x_1 - x_2$$

$$y = \frac{1}{2} \ln \frac{q_1 + q_2}{q_1 - q_2}$$

$$\underset{\text{parton}}{\approx} \frac{1}{2} \ln \frac{x_1}{x_2}$$



- maximal analyzing power of the Born level  $q\bar{q}$ -annihilation subprocess:

$$\hat{G}^{q\bar{q} \rightarrow l^+l^-} \sim \left( | \begin{array}{c} \nearrow \\ \swarrow \end{array} |^2 + | \begin{array}{c} \swarrow \\ \nearrow \end{array} |^2 \right)$$

$$\Delta \hat{G}^{q\bar{q} \rightarrow l^+l^-} \sim \left( | \begin{array}{c} \nearrow \\ \swarrow \end{array} |^2 - | \begin{array}{c} \swarrow \\ \nearrow \end{array} |^2 \right)$$

$$\Rightarrow \Delta \hat{G}^{q\bar{q} \rightarrow l^+l^-} = - \hat{G}^{q\bar{q} \rightarrow l^+l^-}$$

hadron asymmetry:  $\frac{\Delta G}{G} \sim \mathcal{O}\left(\frac{\Delta q}{q}, \frac{\Delta \bar{q}}{\bar{q}}\right) \sim \mathcal{O}(A, \frac{\Delta \bar{q}}{\bar{q}})$

BUT: QCD corrections to the unpolarized

Drell-Yan process are large

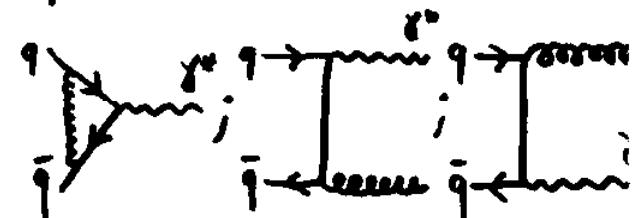
(up to 50% at fixed target energies)

⇒ extraction of  $\Delta \bar{q}(x, Q^2)$  from a measurement of the polarized Drell-Yan process is only reliable at NLO

$dM^2 dx_F$  $dM^2 dy$ 

(Nucl. Phys. B)

- follows the unpolarized calculation of Altarelli, Ellis and Martinelli, NP B157, 461
- work in MS-scheme and use HVBM t'Hooft, Veltman  
NP B44, 183  
Bjorken, Drell  
Comm. Math. Phys. 5 prescription for  $\gamma_s$  in dimensional regularization
- disregard angular distribution of final state leptons  $\rightarrow$  calculate  $\gamma^*$  production

subprocesses:  $O(\alpha_s) - q\bar{q}$ : $O(\alpha'_s) - q\bar{q}$ : $O(\alpha') - gg$ :

$$\begin{aligned}
\frac{dM^2 dx_F}{dM^2 dx_F} &= \frac{9M^2 S}{dM^2 dx_F} \sum_i e_i^2 \int_{x_1^0}^{x_1^0} dx_1 \int_{x_2^0}^{x_2^0} dx_2 \\
&\times \left\{ \left[ \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(0)}}{dM^2 dx_F}(x_1, x_2) + \frac{\alpha_s}{2\pi} \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dx_F} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) \right] \{ \Delta q_i(x_1, \mu_F^2) \Delta \bar{q}_i(x_2, \mu_F^2) \right. \\
&\quad \left. + \Delta \bar{q}_i(x_1, \mu_F^2) \Delta q_i(x_2, \mu_F^2) \} \right. \\
&\quad \left. + \left[ \frac{\alpha_s}{2\pi} \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dx_F} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) \Delta G(x_1, \mu_F^2) (\Delta q_i(x_2, \mu_F^2) + \Delta \bar{q}_i(x_2, \mu_F^2)) \right] \right. \\
&\quad \left. +(1 \leftrightarrow 2) \right\},
\end{aligned}$$

with

$$\begin{aligned}
\frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(0)}}{dM^2 dx_F}(x_1, x_2) &= -\frac{\delta(x_1 - x_1^0) \delta(x_2 - x_2^0)}{x_1^0 + x_2^0} = -\frac{d\hat{\sigma}_{q\bar{q}}^{(0)}}{dM^2 dx_F}(x_1, x_2), \\
\frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dx_F} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) &= -C_F \left\{ \frac{\delta(x_1 - x_1^0) \delta(x_2 - x_2^0)}{x_1^0 + x_2^0} \left[ \frac{\pi^2}{3} - 8 + 2\text{Li}_2(x_1^0) + 2\text{Li}_2(x_2^0) \right. \right. \\
&\quad \left. + \ln^2(1 - x_1^0) + \ln^2(1 - x_2^0) + 2 \ln \frac{x_1^0}{1 - x_1^0} \ln \frac{x_2^0}{1 - x_2^0} \right] \\
&\quad + \left( \frac{\delta(x_1 - x_1^0)}{x_1^0 + x_2^0} \left[ \frac{1}{x_2} - \frac{x_2^0}{x_2^2} - \frac{x_2^{0.2} + x_2^2}{x_2^2(x_2 - x_2^0)} \ln \frac{x_2^0}{x_2} \right. \right. \\
&\quad \left. + \frac{x_2^{0.2} + x_2^2}{x_2^2} \left( \frac{\ln(1 - x_2^0/x_2)}{x_2 - x_2^0} \right)_+ + \frac{x_2^{0.2} + x_2^2}{x_2^2} \frac{1}{(x_2 - x_2^0)_+} \right. \\
&\quad \left. \left. \ln \frac{(x_1^0 + x_2^0)(1 - x_1^0)}{x_1^0(x_1^0 + x_2^0)} \right] + (1 \leftrightarrow 2) \right\} \\
&\quad + \frac{\Delta \tilde{G}^A(x_1, x_2)}{[(x_1 - x_1^0)(x_2 - x_2^0)]_+} + \Delta \tilde{H}^A(x_1, x_2) \\
&\quad + \ln \frac{M^2}{\mu_F^2} \left\{ \frac{\delta(x_1 - x_1^0) \delta(x_2 - x_2^0)}{x_1^0 + x_2^0} \left[ 3 + 2 \ln \frac{1 - x_1^0}{x_1^0} + 2 \ln \frac{1 - x_2^0}{x_2^0} \right] \right. \\
&\quad \left. + \left( \frac{\delta(x_1 - x_1^0)}{x_1^0 + x_2^0} \frac{x_2^{0.2} + x_2^2}{x_2^2} \frac{1}{(x_2 - x_2^0)_+} + (1 \leftrightarrow 2) \right) \right\} \\
&= -\frac{d\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dx_F} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right), \\
\frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dx_F} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) &= -T_F \left\{ \frac{\delta(x_2 - x_2^0)}{(x_1^0 + x_2^0)x_1^2} \left[ (2x_1^0 - x_1) \ln \frac{(x_1^0 + x_2^0)(1 - x_2^0)(x_1 - x_1^0)}{x_1^0 x_2^0 (x_1 + x_2^0)} \right. \right. \\
&\quad \left. + 2(x_1 - x_1^0) \right] + \frac{\Delta \tilde{G}^C(x_1, x_2)}{(x_2 - x_2^0)_+} + \Delta \tilde{H}^C(x_1, x_2) \\
&\quad \left. + \ln \frac{M^2}{\mu_F^2} \left\{ \frac{\delta(x_2 - x_2^0)}{(x_1^0 + x_2^0)x_1^2} (2x_1^0 - x_1) \right\} \right\}.
\end{aligned}$$

where:  $x_F = x_1^0 - x_2^0$ ;  $\tau = x_1^0 x_2^0$   $\Rightarrow x_{12}^0 = \frac{1}{2} (\pm x_F + \sqrt{x_F^2 + 4\tau})$

$$\begin{aligned}\frac{d\Delta\sigma}{dM^2 dy} &= \frac{4\pi\alpha^2}{9M^2 S} \sum_i e_i^2 \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 \\ &\times \left\{ \left[ \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(0)}}{dM^2 dy}(x_1, x_2) + \frac{\alpha_s}{2\pi} \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dy} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) \right] \{ \Delta q_i(x_1, \mu_F^2) \Delta \bar{q}_i(x_2, \mu_F^2) \right. \\ &\quad + \Delta \bar{q}_i(x_1, \mu_F^2) \Delta q_i(x_2, \mu_F^2) \} \\ &\quad \left. + \left[ \frac{\alpha_s}{2\pi} \frac{d\Delta\hat{\sigma}_{q\bar{q}}^{(1)}}{dM^2 dy} \left( x_1, x_2, \frac{M^2}{\mu_F^2} \right) \Delta G(x_1, \mu_F^2) \{ \Delta q_i(x_2, \mu_F^2) + \Delta \bar{q}_i(x_2, \mu_F^2) \} \right. \right. \\ &\quad \left. \left. + (1 \leftrightarrow 2) \right] \right\}.\end{aligned}$$

agrees with:  
Weber: NP 3322,

with:  $\gamma = \frac{1}{2} \ln \frac{x_1^0}{x_2^0}; \tau = x_1^0 x_2^0 \Rightarrow x_{1,2}^0 = \sqrt{\tau} e^{\pm i\gamma}$

## Drell-Yan invariant mass distribution:

$$\begin{aligned}\frac{d\Delta\sigma}{dM^2} &= \frac{4\pi\alpha^2}{9SM^2} \int_0^1 dx_1 dx_2 dz \delta(x_1 x_2 z - \tau) \sum_q e_q^2 \\ &\times \left\{ \{ \Delta q(x_1, \mu_F^2) \Delta \bar{q}(x_2, \mu_F^2) + (1 \leftrightarrow 2) \} \left( \Delta c_{q\bar{q}}^{DY,(0)}(z) + \frac{\alpha_s}{2\pi} \Delta c_{q\bar{q}}^{DY,(1)} \left( z, \frac{M^2}{\mu_F^2} \right) \right) \right. \\ &\quad \left. + \{ (\Delta q(x_1, \mu_F^2) + \Delta \bar{q}(x_1, \mu_F^2)) \Delta G(x_2, \mu_F^2) + (1 \leftrightarrow 2) \} \frac{\alpha_s}{2\pi} \Delta c_{q\bar{q}}^{DY,(1)} \left( z, \frac{M^2}{\mu_F^2} \right) \right\},\end{aligned}$$

with:

$$\begin{aligned}\Delta c_{q\bar{q}}^{DY,(0)}(z) &= -\delta(1-z) = -c_{q\bar{q}}^{DY,(0)}(z) \\ \Delta c_{q\bar{q}}^{DY,(1)} \left( z, \frac{M^2}{\mu_F^2} \right) &= -C_F \left[ 8 \left( \frac{\ln(1-z)}{1-z} \right)_+ - 2 \frac{1+z^2}{1-z} \ln z - 4(1+z) \ln(1-z) \right. \\ &\quad + \delta(1-z) \left( -8 + \frac{2\pi^2}{3} \right) \\ &\quad \left. + 2 \ln \frac{M^2}{\mu_F^2} \left\{ \left( \frac{2}{1-z} \right)_+ - 1 - z + \frac{3}{2} \delta(1-z) \right\} \right], \\ &= -c_{q\bar{q}}^{DY,(1)} \left( z, \frac{M^2}{\mu_F^2} \right) \\ \Delta c_{q\bar{q}}^{DY,(1)} \left( z, \frac{M^2}{\mu_F^2} \right) &= -T_F \left[ (2z-1) \ln \frac{(1-z)^2}{z} + \frac{5}{2} - z - \frac{3}{2} z^2 + \ln \frac{M^2}{\mu_F^2} \{ 2z-1 \} \right].\end{aligned}$$

agrees with:

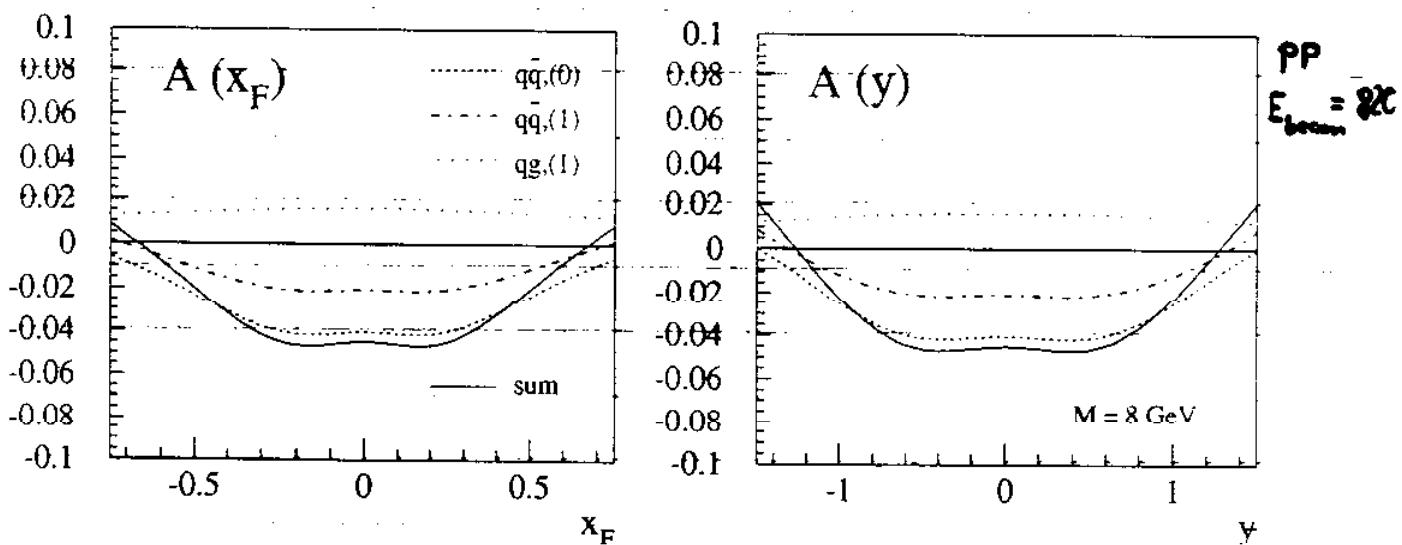
Rakotobe: NP 3323,  
Weber: NP 3322,  
Kowal: PR 353, II

Consider:

$$A(x_F) = \frac{d\Delta G}{dM dx_F} / \frac{dG}{dM dx_F} \text{ and } A(y) = \frac{d\Delta G}{dM dy} / \frac{dG}{dM dy}$$

- with polarized GS(A) and unpolarized MRS(A) parton distributions,  $\mu_F = M$ .

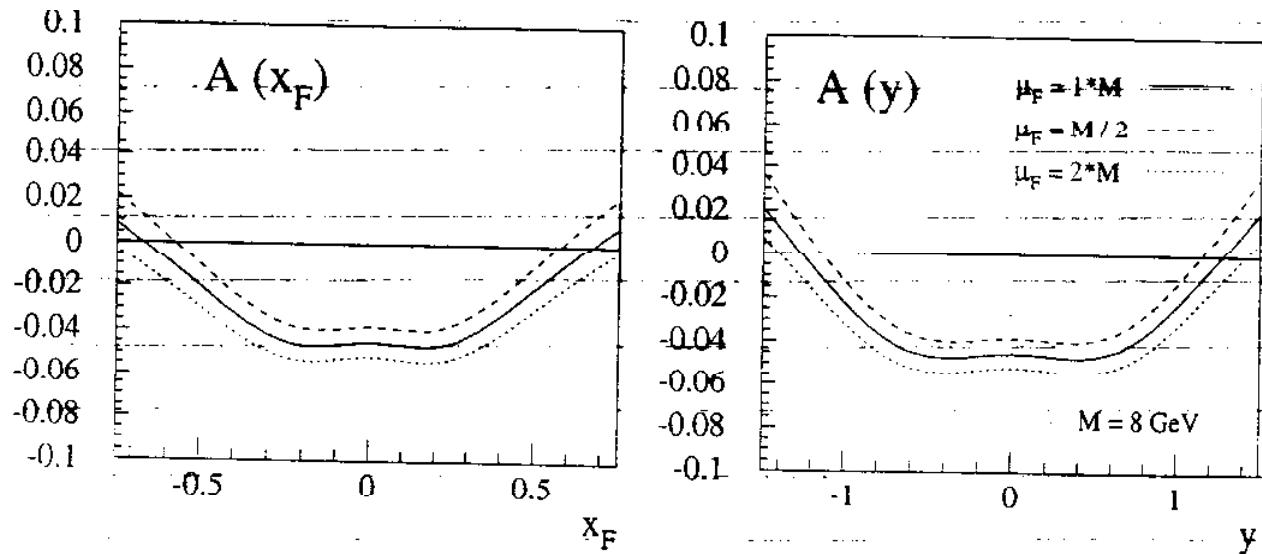
Contributions of the individual subprocesses normalized to the full unpol. cross section



- $q\bar{q}^{(0)}$  and  $q\bar{q}^{(1)}$  contribute with the same sign
- sign of  $qg^{(1)}$  contribution opposite to  $q\bar{q}^{(0,1)}$
- uncertainty on  $\Delta G(x, Q^2)$  at large  $x$  prevents conceivable contradiction of  $K$ -factor

mass factorization scale  $\mu_F$ :

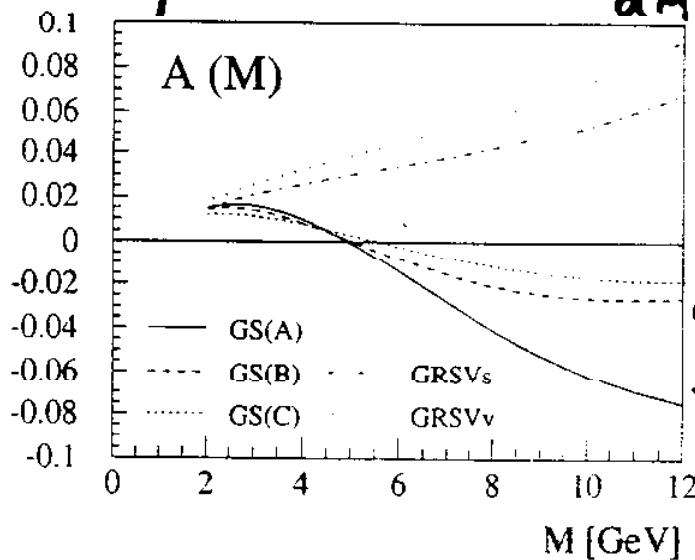
pp,  $E_{beam} = 820 \text{ GeV}$ , GS(A), MRS(A')



Variation of  $A$  for  $0.5M \leq \mu_F \leq 2M$   
estimates the theory error to be  
 $\pm 0.01$  compared to an asymmetry of  $O(0.05)$

Mass dependence of  
the asymmetry:

$$A(M) = \frac{\frac{d\Delta G}{dM}}{\frac{dG}{dM}}$$

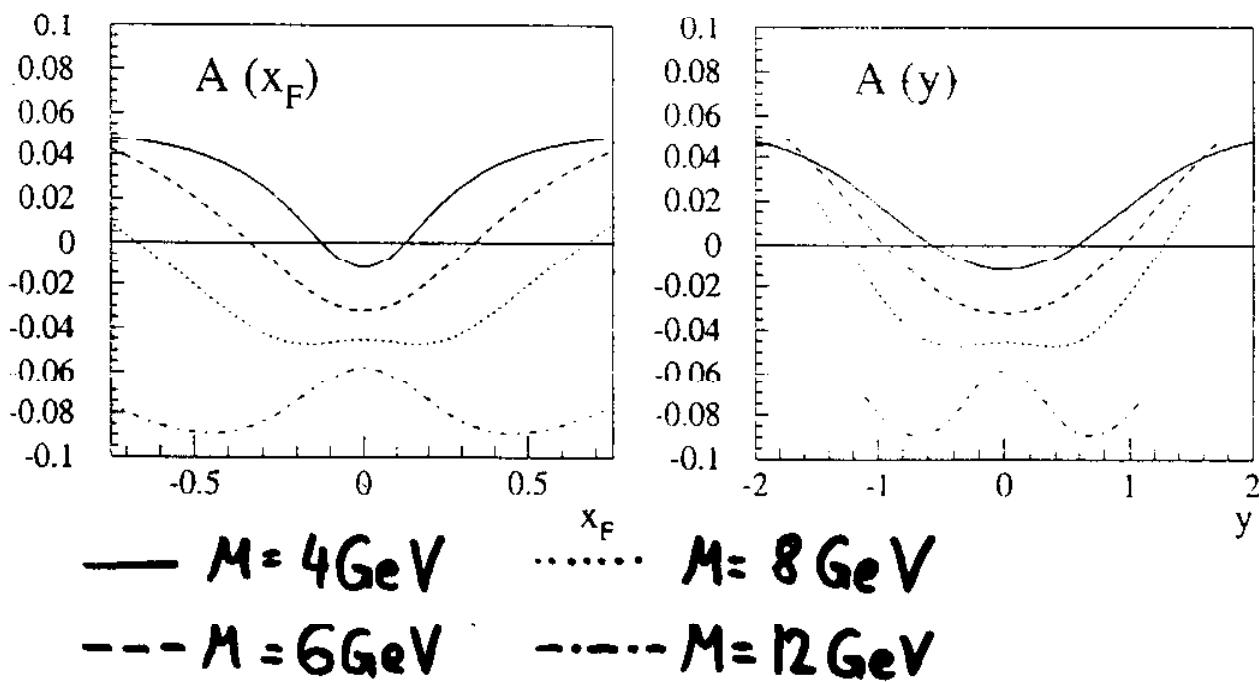


pp - collisions

$E_{beam} = 820 \text{ GeV}$

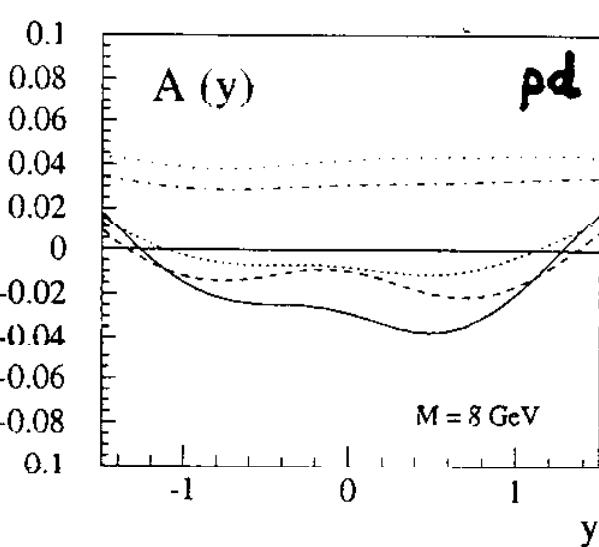
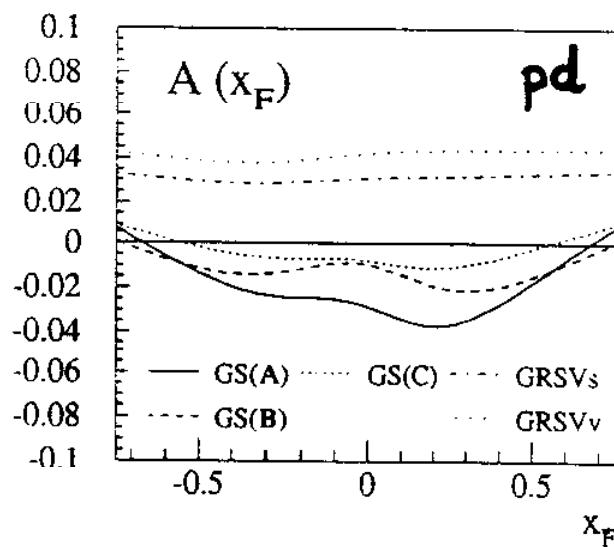
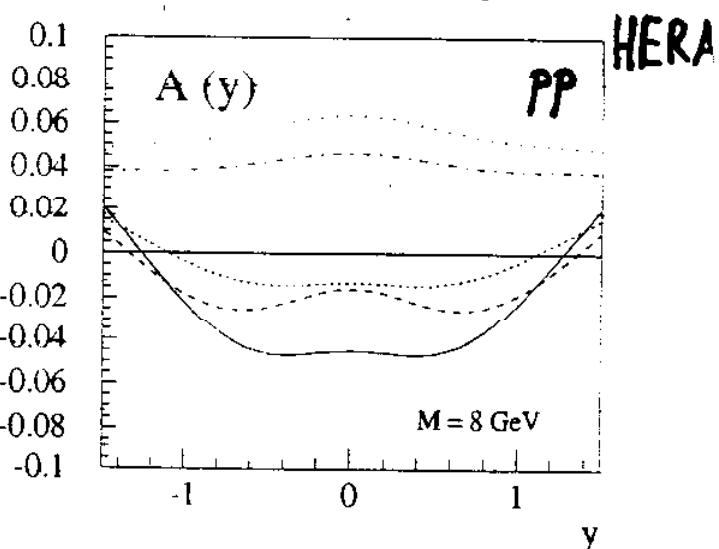
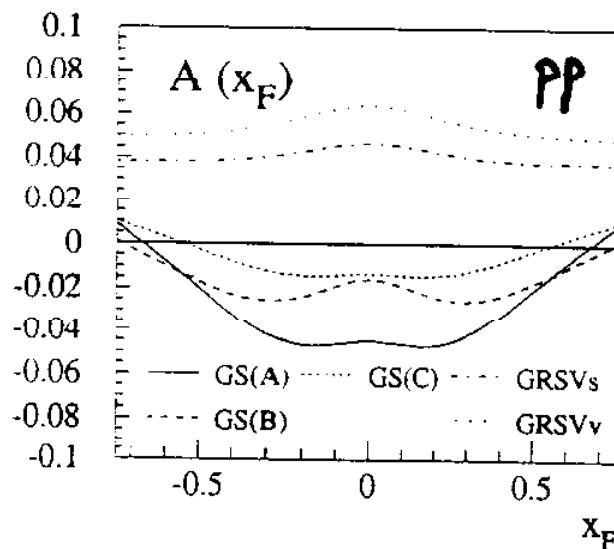
Parton distributions:  
Gläck, Reya, Staudenmaier,  
PR D53, 4775  
T6, Stirling,  
PR D53, 6100

$x_F$ - and  $y$ -distributions for different  $M^2$   
(polarized GS(A) parton distributions)



Unpolarized measurements (CERN-NASl, Fermilab-E866) of the Drell-Yan cross section in pp and pd collisions probe light quark sea:  $\bar{u}(x, Q^2) - \bar{d}(x, Q^2)$ .

Expected asymmetries in polarized pp and pd collision



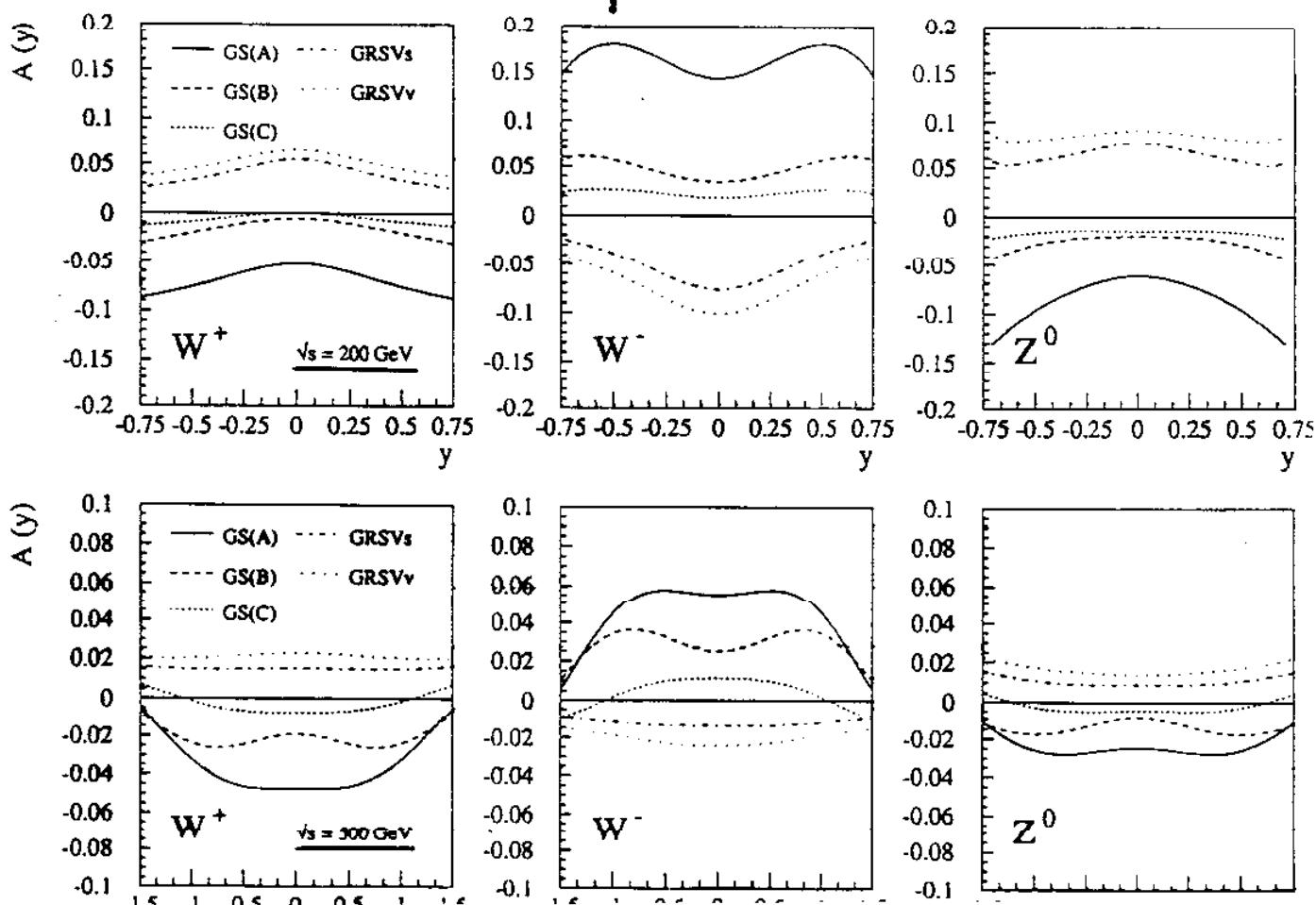
! high statistics required to measure  $\Delta\bar{u}(x, Q^2) - \Delta\bar{d}(x, Q^2)$

- Production of  $W^\pm, t$  in hadronic collisions is similar to Drell-Yan process.
- QCD corrections to Drell-Yan process for  $M=M_W, Z$  and to vector boson production are identical
- Different vector bosons probe different distributions

$$A(W^+) \sim \frac{\Delta u}{u} \frac{\Delta \bar{d}}{\bar{d}}$$

$$A(W^-) \sim \frac{\Delta d}{d} \frac{\Delta \bar{u}}{\bar{u}}$$

$$A(Z^0) \sim \frac{\Delta g}{g}$$



- Complete  $\mathcal{O}(\alpha_s)$  QCD corrections to the  $x_F$ - and  $y$ -distributions of Drell-Yan pairs in collisions of longitudinally polarized hadrons are now available.
- Magnitude of corrections similar to unpolarized Drell-Yan process; K-factor uncertain due to unknown  $\Delta G(x, Q^2)$  at large  $x$
- Scale uncertainty (theory error) on Drell-Yan asymmetry is  $\mathcal{O}(0.01)$  at NLO.
- Results can be applied to lepton pair production at fixed target experiments (HERA- $\bar{N}$ ) and to vector boson production at RHIC.